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Common fixed point theorems for non-self-mappings in metric spaces of hyperbolic type

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ABSTRACT

In this paper, the concept of a pair of non-linear contraction type mappings in a metric space of hyperbolic type is introduced and the conditions guaranteeing the existence of a common fixed point for such non-linear contractions are established. Presented results generalize and improve some of the known results. An example is constructed to show that our theorems are genuine generalizations of the main theorems of Assad, Ćirić, Khan et al., Rhoades and Imdad and Kumar. One of the possible applications of our results is also presented.

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1. Introduction

Fixed point theorems for contraction self-mappings have find applications in diverse disciplines of mathematics, engineering and economics. In convex spaces occur cases where the involved function is not necessarily a self-mapping of a closed subset. Assad [1] and Assad and Kirk [2] first studied non-self-contraction mappings in a metric space (X, d), metrically convex in the sense of Menger (that is, for each x, y in X with $x \neq y$ there exists z in X, $x \neq z \neq y$, such that $d(x, z) + d(z, y) = d(x, y)$). In recent years, this technique has been developed and fixed and common fixed points of non-self-mappings have been studied by many authors [3–14]. Some of the obtained results have find applications (c.f. [2,14–16]). In numerical mathematics, a restricted condition $T(\partial K) \subseteq K$ is especially favorable instead of $T(K) \subseteq K$, where K is a closed subset of X, $T : K \to X$ and ∂K is the boundary of K.

In an attempt to generalize a theorem of Assad [1] and Assad and Kirk [2], Rhoades [13] proved the following result in a Banach space.

Theorem 1. *Let X be a Banach space, K a non-empty closed subset of X and $T : K \to X$ a mapping of K into X satisfying the condition*

$$d(Tx, Ty) \leq h \max\left\{\frac{d(x, y)}{2}, d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{q}\right\} \tag{1}$$

for all x, y in K, $0 < h < 1$, $q \geq 1 + 2h$ and T has the additional property that for each $x \in \partial K$, the boundary of K, $Tx \in K$, then T has a unique fixed point.

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